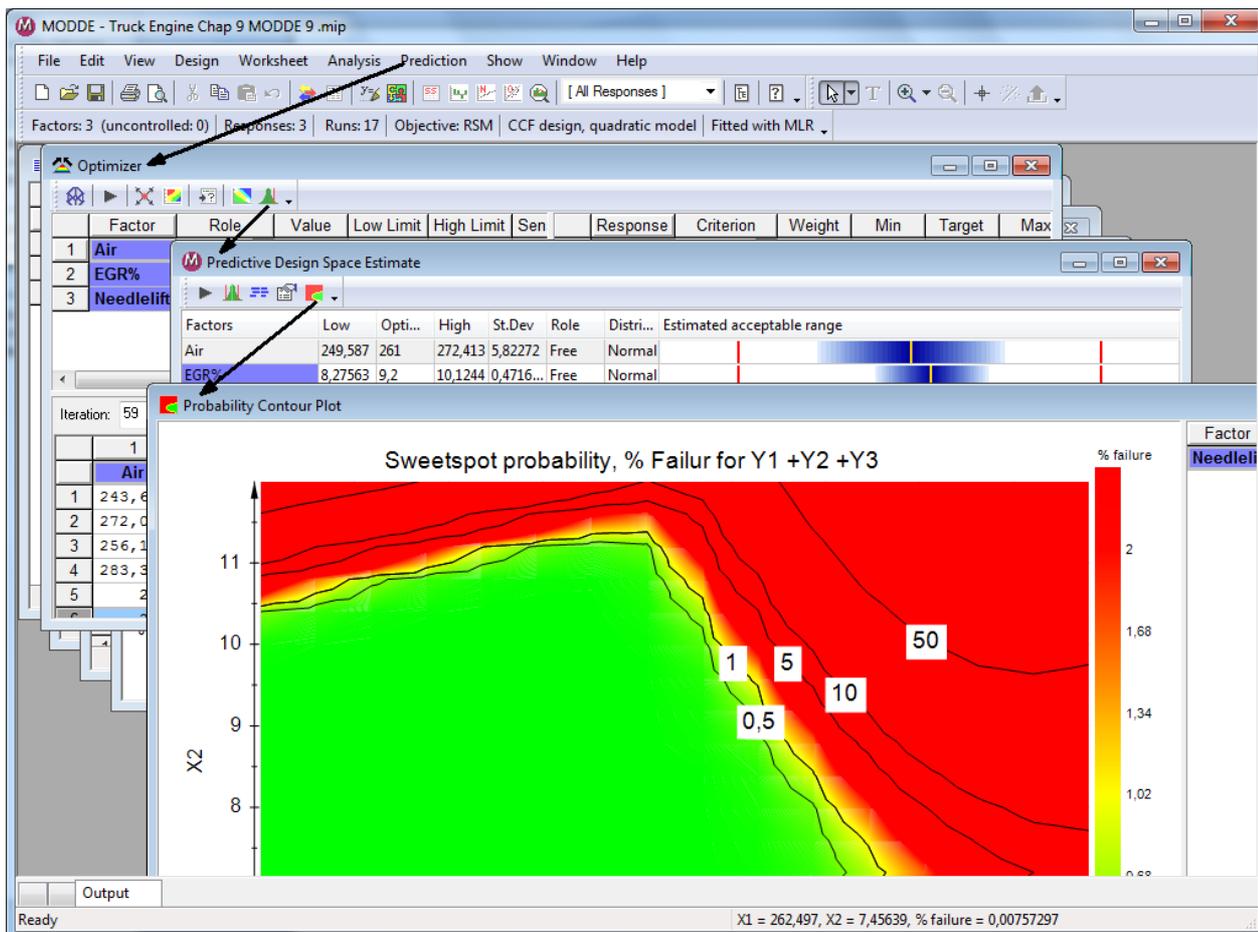# Probability Contour Plot

Using MODDE for Design of Experiments

Creating a Probability Contour Plot in MODDE 9.1.

There are two main objectives;

1) Create a contour plot or a sweet spot plot that consider the model prediction error.
2) Create a contour or a sweet spot plot that include the uncertainty in the used factors as well. This plot can be interpreted as a Design Space.

The function Probability Contour Plot is reached from the Optimizer/Design Space Estimation function. It is important that models are evaluated and optimized according to standard procedures. Inherently, the prediction error depends on the model quality. There is no absolute demand that R2/Q2 is high, the important part is that the model represents the data at hand in the best possible way.

The example used in this tutorial comprise 3 factors and 3 responses, the data originates from Volvo Technology Report, #2, 1997, with PI Sven Ahlinder. To make the case illustrative for the purpose of demonstrating the possibilities with the new Probability Contour Plot functionality in MODDE 9.1 changes have been made in specifications that shall not be considered as application relevant. Therefore the factors will be named X1 – X3 and the responses Y1 – Y3.

Case 1

Create a contour plot or a sweet spot plot that consider the model prediction error.

The classical contour plot as displayed in Figure 1 shows the average prediction (point estimate prediction) for every combination of X1 and X2 settings with all other factors at a constant level. There is no information of prediction error in this type of presentation.
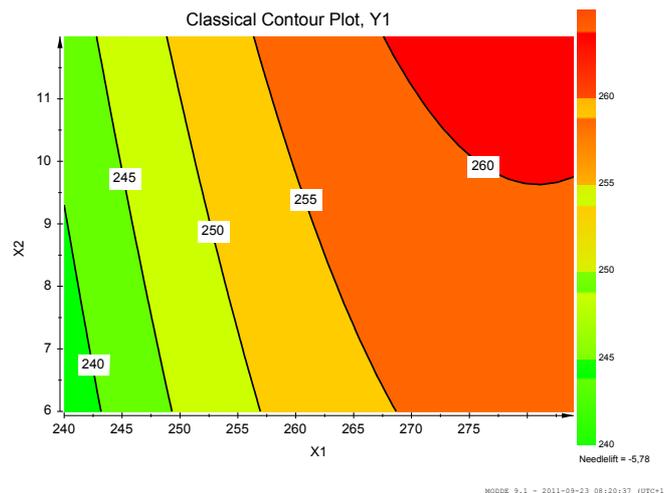

Consider a specification for Y1 that all results < 260 are acceptable; the combinations of factors X1 and X2 that give predictions close to the contour line 260 increase the probability to get a value > 260. The risk of not meeting the specification Y1 < 260 with production settings close to the border increases. To help the user assess this risk the corresponding plot can be made as a probability of meeting the specification. The plot is named Probability Contour Plot (figure 2). In this plot the contour lines represent the risk of not meeting the specification with various settings of X1 and X2.
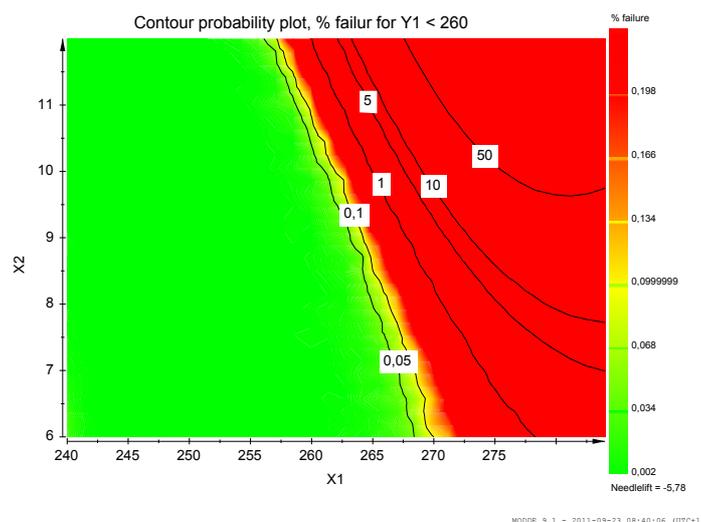
The Probability Contour Plot shows that the risk of getting a value > 260 for Y1 reaches 50% on the classical contour plot line 260 (figure 1 and 2). The color coding is according an acceptable risk of 0.1% of obtaining a prediction > 260. This reduces the acceptable region significantly and it will give the user a more realistic interpretation of the possibilities and limitations.
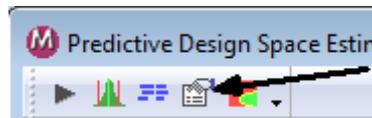
The Probability Contour Plot is created from the Design Space Estimation tool.
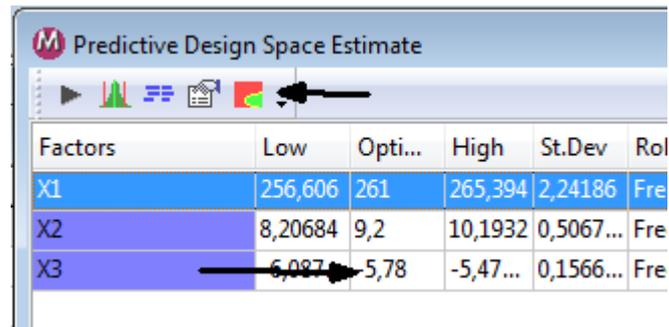
Factors not used in the contour plot need to have a set value, e.g. an optimal set point chosen by the optimizer. The acceptable risk level is specified in DPMO (defects per one million opportunities) in Properties and is entered in the field for DPMO target (1000 DPMO = 0.1% failure).

Open Properties

Run the Probability contour function
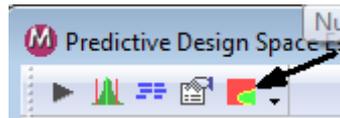
Follow the contour plot wizard to select factors and responses to be displayed in the plot. If several responses are selected a sweet spot plot will be generated. In Plot Options the plot resolution can be set. The results are based on Monte Carlo simulations at a default grid of 16*16 points in the plot surface and the number of simulations at each point is default 10 000. Increasing the number of simulations and number of points (e.g. 32*32) will significantly increase the time required for creation of the plot.
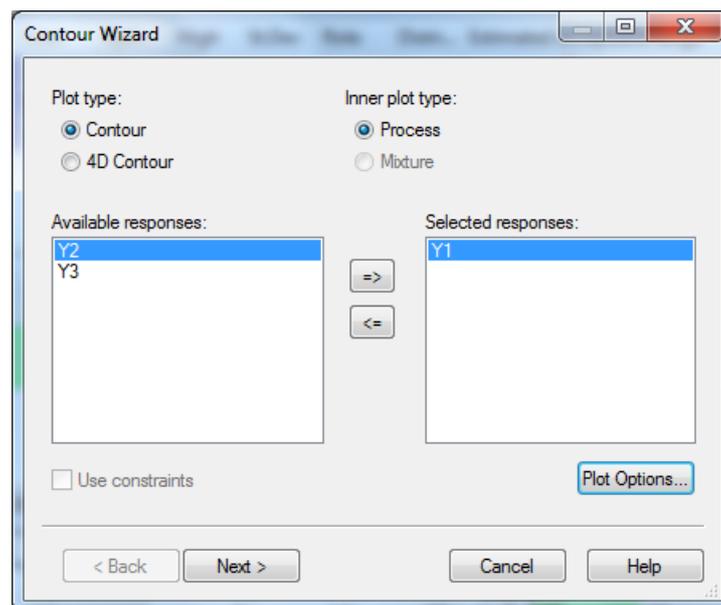

In Plot Options it is also possible to specify the measure of risk which will be displayed in the Probability Contour Plot. There are two options; DPMO (default) or %.

Step 2 in the wizard has 2 checkboxes; these should be left empty to present a model error probability plot. This function will be discussed in Case 2.

☐ Include distribution on axis factors

☐ Include distribution on constant factors

This will generate a Probability Contour Plot for Y 1 which considers only the model prediction error.
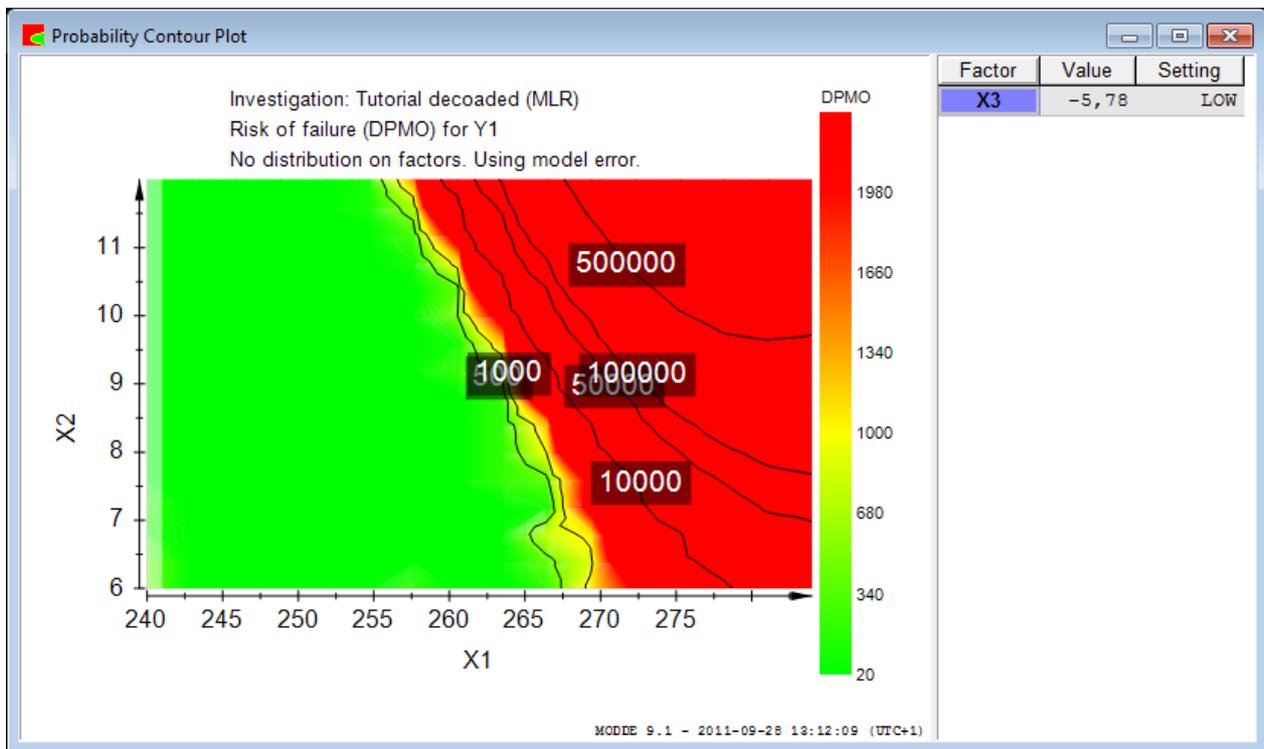
Case 2

Create a contour or a sweet spot plot that includes the uncertainty in the used factors as well (Design Space).

In most applications a specified factor setting means that it will be the required value +/- something. As an example the best pH level for a studied application was found to be 6.2. But in normal operations pH=6.2 means 6.2 +/- 0.15 because that's the current precision in the equipment used. To retrieve a good estimation of result values that may be expected from an investigated system those parts of the systematic error have to be accounted for as well.

There are two sources of uncertainty that can be considered when estimating a region with low risk of not meeting the specifications; model error and the precision in factor setting. In Case 1 only the model error was discussed.

To illustrate the influence of factor variability, the following is imposed in Case 2:

X1 = +/- 4.0

X2 = +/- 0.2

X3 = +/- 0.1

This will probably give a better estimate of the future outcome of the process. These settings are specified in the Design Space Estimation window, as an interval Low to High or as a Standard deviation around the Optimum. The Low to High range is default a 95% range of a normal distribution.

When the Probability Contour Plot is calculated some factors will not be presented (depending on the number of factors and dimensions selected for the plot). These factors will be set to the "optimal" value but the distribution around that constant setting will be used in the presentation.

This function is enabled in the second page of the wizard.


The probability contour plot will now present a region where the risk of failure is less than 0.1% for response Y1. The area colored in green represent factor settings with less than 0.1% risk of yielding a value for Y1 outside of specification.
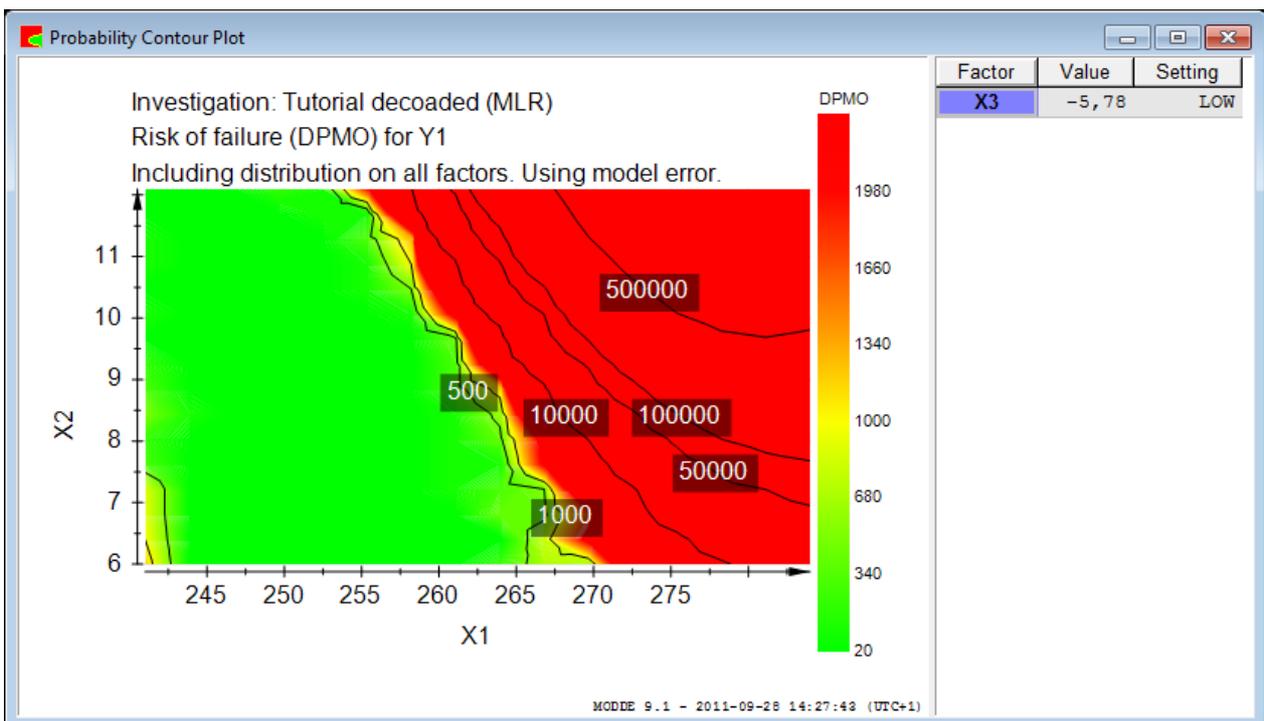
The probability contour plot is presented as a Sweet spot plot with several responses and the corresponding specifications.
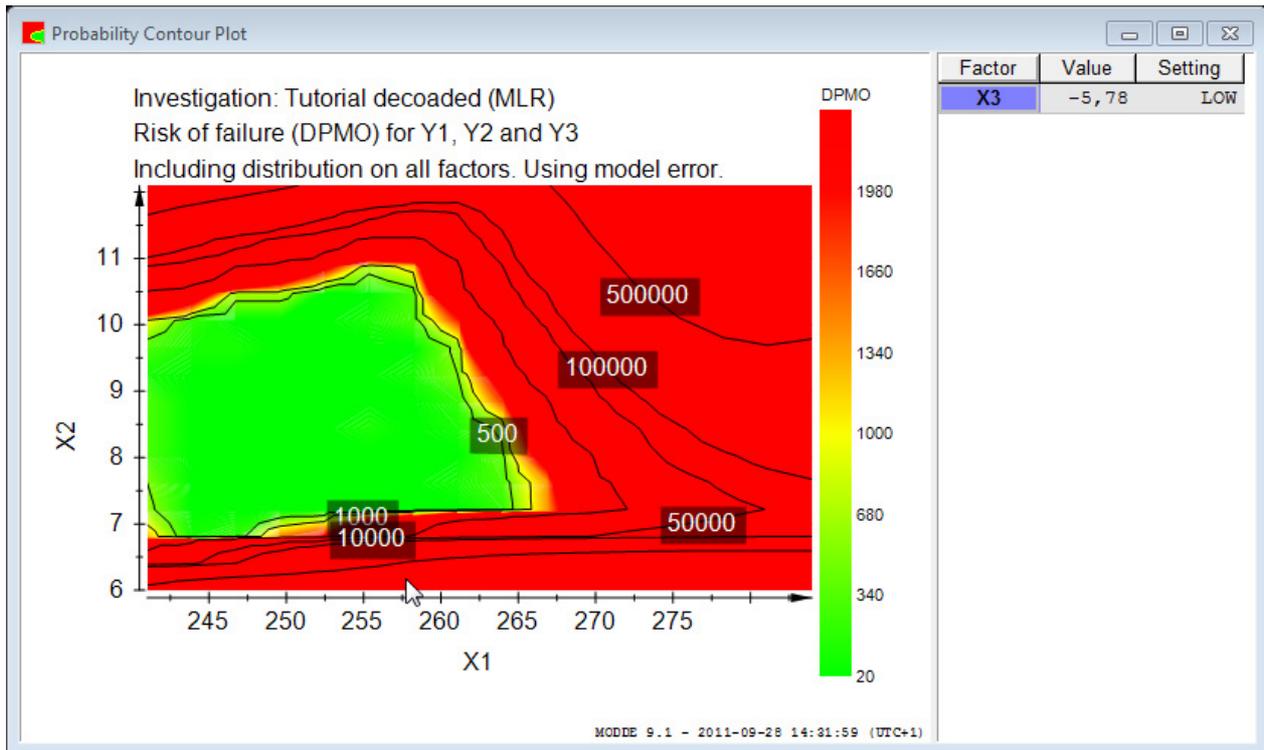
The generated a Probability Contour Plot considers the model prediction error and uncertainty in the used factors for all included responses.

www.umetrics.com
info@umetrics.com