Controlling a Pendulum with VisSim/OptimizePRO

Problem: Determine the least amount of energy used to drive a pendulum to a specific final position within a prescribed time and tolerance.

The position of the pendulum is specified as angle Θ , between $-\pi$ and π. Thus, when $\Theta = 0$, the pendulum is positioned at 6 o'clock; when $\Theta = \pm\,\pi$, its positioned at 12 o'clock.

To begin, assume the pendulum is in starting position $\Theta = 0$, and the desired final position is determined by the value of the variable Setpoint. In this case, we use Setpoint = $\pi/2$, which means we want to drive the pendulum from its 6 o'clock position to 3 o'clock.

The closed-loop control U has the form:

$$U = A\, sin(\Theta - S) + B\, \Theta' + (g/l) sin(S)$$

where:

A and B determine the energy used to drive the pendulum to its final position

$A\, sin(\Theta - S)$, $B\, \Theta'$ provide the feedback information

$(g/l) sin(S)$ balances out the effect of gravity at the setpoint

In determining optimal settings for A and B, we use a time period of 10 seconds and a tolerance of ± 0.01 for the final pendulum position. Analytically, the equation for this system is:

$$M\Theta'' = -(g/l) sin(\Theta) + U$$

where:

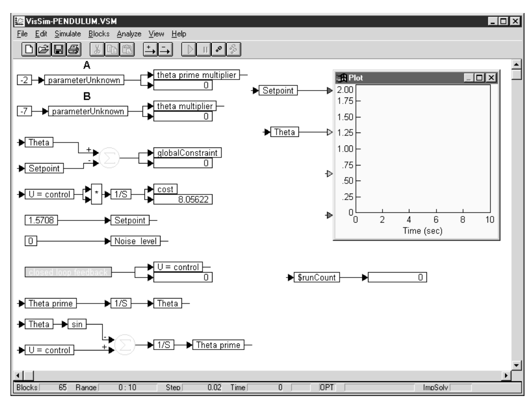
$$U = A\, sin(\Theta - S) + B\, \Theta' + (g/l) sin(S)$$

In the above equation, S represents the setpoint with the following boundary conditions:

$$\Theta(0) = \Theta'(0) = 0$$

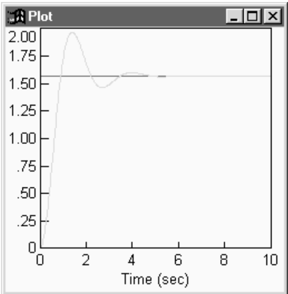
$$S - .01 <= \Theta(10) <= S + .01$$

For simplicity, we can assume that the length of the pendulum and the gravity constant cancel out (that is, $g/l = 1$) and the mass of the pendulum is $M = 1$.



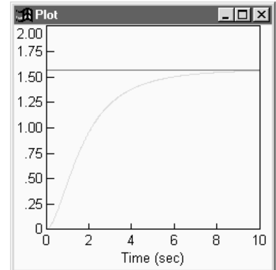
The VisSim diagram (right) illustrates what we're trying to accomplish. The two parameterUnknown blocks represent A and B; the single globalConstraint block represents the pendulum's final position tolerance; and the cost block represents the energy used.

The initial parameterUnknown settings are $A = -2$ and $B = -7$. A simulation run with these settings moves the pendulum to 3 o'clock with a cost of 27.1.



The plot (left) shows the path of the angle of the pendulum from 0 to $\pi/2$. These parameter settings cause the pendulum to overshoot the target and thus use more energy than necessary.

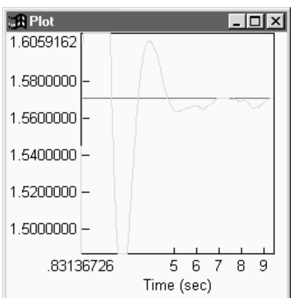
An optimization run using VisSim/OptimizePRO returns *A* = -2.61 and *B* = -1 with a corresponding cost of 8.1. This time, the plot (right) shows that the pendulum takes a smooth path to its final position, does not overshoot the target, and uses less energy.



Applying Real-World Conditions

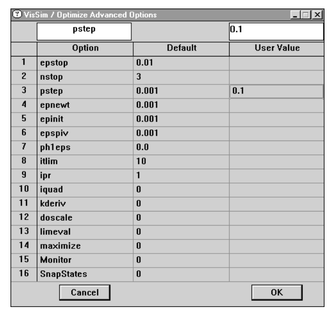
In practice, measurements are always subject to error. In order to demonstrate the effect of measurement errors on the optimization process, the diagram includes a variable (Noise level) that controls the error in measurement of Q. When Noise level is 0, there is no error in Q, when it is 0.05, the measurement error is 5%, and so on.

Noise is added to the value of Q via the Closed Loop Feedback compound block (highlighted in blue). Noise is produced by a gaussian block with a mean of 0 and standard deviation of 0.33. A simulation run with Noise level = 0.05 and parameterUnknown settings A = -2 and B = -7 moves the pendulum to 3 o'clock with a cost of 27.3. However, due to the high noise level, the optimization run initially fails to improve the cost, as shown in the plot on the left.



The results are almost identical to those achieved when Noise level = 0. In addition, an error message in the report file states:

FAILURE: Function values appear to be noisy and nonsmooth



The noise in Q is responsible for preventing the optimization process from making any improvement in the cost. To improve the situation, we can reset VisSim/OptimizePRO's pstep option to 0.1 (by default it is set to 0.001). This setting causes the optimizer to take a larger step when it computes derivatives. As a rule of thumb, the pstep should be larger than the noise level. A final optimization run with Noise level = 0.05 and pstep = 0.1 returns *A* = -8.7 and *B* = -4.0 with a cost of 8.8, as shown in the plot on the right.

This time, VisSim/OptimizePRO succeeds in minimizing the cost while determining optimal settings for *A* and *B*.